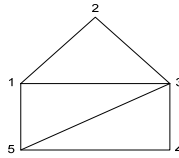


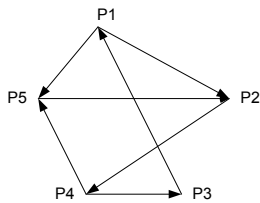
# Graphs

1.
  - (a) How many binary relations are there on a set  $A$  with  $n$  elements?
  - (b) How many one-to-one functions are there from a set  $A$  with  $n$  elements onto itself?
  - (c) Show that the number of odd degree vertex in a finite graph is even.
  - (d) Specify an adjacency-lists representation of the undirected graph.

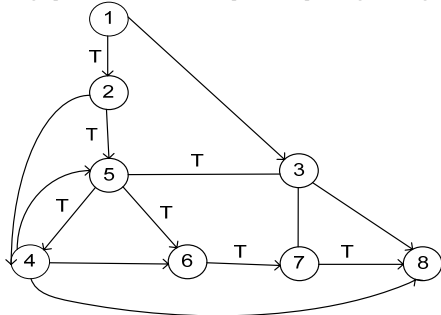


2. Answer the following question briefly.
  - (i) If the transportation problem is solved using some version of the simplex algorithm, under what conditions will the solution always have integer values?

3. Write the adjacency matrix representation of the graph given in fig below:

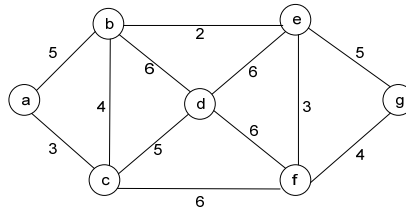


4. In the graph shown below, the depth-first spanning tree edges are marked with 'T'. Identify the forward, backward and cross edges.



5. The maximum number of possible edges in an undirected graph with  $n$  vertices and  $k$  components is \_\_\_\_\_.
6. Show that all vertices in an undirected finite graph cannot have distinct degrees, if the graph has at least two vertices
7. Maximum number of edges in a planar graph with  $n$  vertices is \_\_\_\_\_.
8. Consider a simple connected graph  $G$  with  $n$  vertices and  $n$  edges ( $n > 2$ ). Then which of the following statements are true
  - (a)  $G$  has no cycles
  - (b) the graph obtained by removing any edges from  $G$  is not consider connected
  - (c)  $G$  has at least one cycle
  - (d) the graph obtained by removing any two edges from  $G$  is not consider connected
  - (e) none of the above.

9. Consider the following graph:



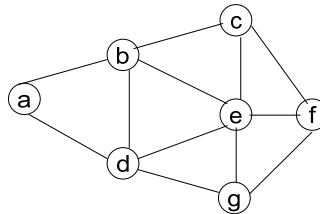
Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm?

- (A) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)
- (B) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)
- (C) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)
- (D) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

10. Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?

- (A) No two vertices have the same degree.
- (B) At least two vertices have the same degree.
- (C) At least three vertices have the same degree.
- (D) All vertices have the same degree.

11. Consider the following sequences of nodes for the undirected graph given below.

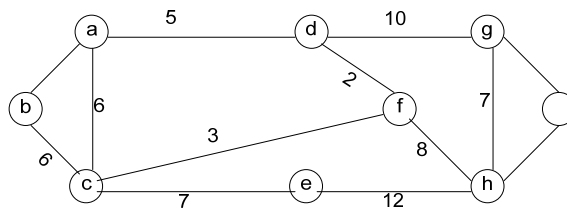


- I. a b e f d g c
- II. a b e f c g d
- III. a d g e b c f
- IV. a d b c g e f

A Depth First Search (DFS) is started at node *a*. The nodes are listed in the order they are first visited. Which all of the above is (are) possible output(s)?

- (A) I and III only
- (B) II and III only
- (C) II, III and IV only
- (D) I, II and III only

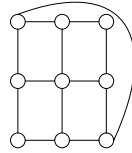
12. For the undirected, weighted graph given below, which of the following sequences of edges represents a correct execution of Prim's algorithm to construct a Minimum Spanning Tree?



- (D) (a, b), (d, f), (f, c), (g, i), (d, a), (g, h), (c, e), (f, h)
- (E) (c, e), (c, f), (f, d), (d, a), (a, b), (g, h), (h, f), (g, i)
- (F) (d, f), (f, c), (d, a), (a, b), (c, e), (f, h), (g, h), (g, i)
- (G) (h, g), (g, i), (h, f), (f, c), (f, d), (d, a), (a, b), (c, e)

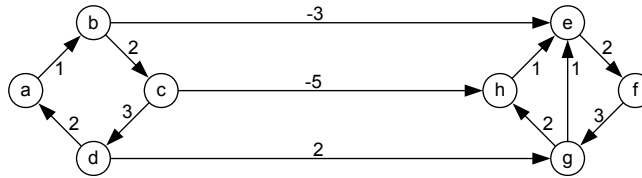
13.  $G$  is a simple undirected graph. Some vertices of  $G$  are of odd degree. Add a node  $v$  to  $G$  and make it adjacent to each odd degree vertex of  $G$ . The resultant graph is sure to be
- (A) regular
  - (B) complete
  - (C) Hamiltonian
  - (D) Euler

14. What is the chromatic number of the following graph?



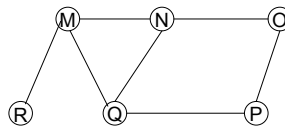
- (A) 2
- (B) 3
- (C) 4
- (D) 5

- 15.



Dijkstra's single source shortest path algorithm when run from vertex  $a$  in the above graph, computes the correct shortest path distance to

- (A) only vertex  $a$
  - (B) only vertices  $a, e, f, g, h$
  - (C) only vertices  $a, b, c, d$
  - (D) all the vertices
16.  $G$  is a graph on  $n$  vertices and  $2n - 2$  edges. The edges of  $G$  can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for  $G$ ?
- (A) For every subset of  $k$  vertices, the induced subgraph has at most  $2k - 2$  edges
  - (B) The minimum cut in  $G$  has at least two edges
  - (C) There are two edge-disjoint paths between every pair of vertices
  - (D) There are two vertex-disjoint paths between every pair of vertices
17. Which of the following statements is true for every planar graph on  $n$  vertices?
- (A) The graph is connected
  - (B) The graph is Eulerian
  - (C) The graph has a vertex-cover of size at most  $3n/4$
  - (D) The graph has an independent set of size at least  $n/3$
18. The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is

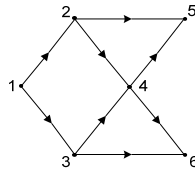


- (A) MNOPQR
- (B) NQMPOR
- (C) QMNPRO
- (D) QMNPOR

19. The most efficient algorithm for finding the number of connected components in an undirected graph on  $n$  vertices and  $m$  edges has time complexity
- (A)  $\Theta(n)$
  - (B)  $\Theta(m)$
  - (C)  $\Theta(m + n)$
  - (D)  $\Theta(mn)$

20. What is the largest  $m$  such that every simple connected graph with  $n$  vertices and  $n$  edges contains at least  $m$  different spanning trees?
- (A) 1 (B) 2 (C) 3 (D)  $n$

21. Consider the DAG with  $V = \{1, 2, 3, 4, 5, 6\}$ , shown below.



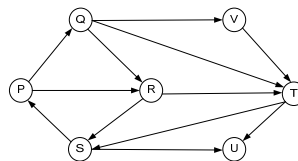
Which one the following is **NOT** a topological ordering?

- (A) 1 2 3 4 5 6
  - (B) 1 3 2 4 5 6
  - (C) 1 3 2 4 6 5
  - (D) 3 2 4 1 6 5
22. Let  $G$  be the non-planar graph with the minimum possible number of edges. Then  $G$  has
- (A) 9 edges and 5 vertices
  - (B) 9 edges and 6 vertices
  - (C) 10 edges and 5 vertices
  - (D) 10 edges and 6 vertices

23. Consider the depth first search of an undirected graph with 3 vertices  $P, Q$  and  $R$ . Let discovery time  $d(u)$  represent the time instant when the vertex  $u$  is first visited, and finis time  $f(u)$  represent the time instant when the vertex  $u$  is last visited. Given that
- $d(P) = 5$  units       $f(P) = 12$  units
- $d(Q) = 6$  units       $f(Q) = 10$  units
- $d(R) = 14$  units       $f(R) = 18$  units

Which of the following statements is TRUE about the graph?

- (A) There is only one connected component
  - (B) There are connected components, and  $P$  and  $R$  are connected
  - (C) There are connected components, and  $Q$  and  $R$  are connected
  - (D) There are connected components, and  $P$  and  $Q$  are connected
24. Which of the following is the correct decomposition of the directed graph given below into its strongly connected components?



- (A)  $\{P, Q, R, S\}, \{T\}, \{U\}, \{V\}$
- (B)  $\{P, Q, R, S, T, V\}, \{U\}$
- (C)  $\{P, Q, S, T, V\}, \{R\}, \{U\}$
- (D)  $\{P, Q, R, S, T, U, V\}$

25. Consider the undirected graph  $G$  defined as follows. The vertices of  $G$  are bit strings of length  $n$ . We have an edge between vertex  $u$  and vertex  $v$  if and only if  $u$  and  $v$  differ in exactly one bit position (in other words,  $v$  can be obtained from  $u$  by flipping a single bit). The ratio of the chromatic number of  $G$  to the diameter of  $G$  is

- (A)  $1/2^{n-1}$  (B)  $1/n$  (C)  $2/n$  (D)  $3/n$

26. If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a

- (A) Hamiltonian cycle (B) grid (C) hypercube (D) tree

27. In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

- (A) 10 (B) 11 (C) 12 (D) 15

28. The  $2^n$  vertices of a graph  $G$  corresponds to all subsets of size  $n$ , for  $n \geq 6$ . Two vertices of  $G$  are adjacent if and only if the corresponding sets intersect in exactly two elements.

The number of connected components in  $G$  is

- (A) 2 (B)  $n+2$  (C)  $2^{n/2}$  (D)  $\frac{2^n}{n}$

The maximum degree of a vertex in  $G$  is

- (A)  $\binom{n/2}{2} 2^{n/2}$  (B)  $2^{n-2}$  (C)  $2^{n-3} \times 3$  (D)  $2^{n-1}$

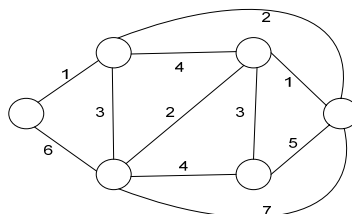
The number of vertices of degree zero in  $G$  is

- (A) 1 (B)  $n$  (C)  $n+1$  (D)  $2^n$

29. Let  $T$  be a depth first search tree in an undirected graph  $G$ . Vertices  $u$  and  $v$  are leaves of this tree  $T$ . The degrees of both  $u$  and  $v$  in  $G$  are at least 2. Which one of the following statements is true?

- (A) There must exist a vertex  $w$  adjacent to both  $u$  and  $v$  in  $G$   
 (B) There must exist a vertex  $w$  whose removal disconnects  $u$  and  $v$  in  $G$   
 (C) There must exist a cycle in  $G$  containing  $u$  and  $v$   
 (D) There must exist a cycle in  $G$  containing  $u$  and  $v$  and all its neighbours in  $G$

30. Consider the following graph



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm?

- (A) (a-b), (d-f), (b-f), (d-c), (d-e) (B) (a-b), (d-f), (d-c), (b-f), (d-e)

(C) (d-f), (a-b), (d-c), (b-f), (d-e)

(D) (d-f), (a-b), (b-f), (d-e), (d-c)

31. To implement Dijkstra's shortest path algorithm on undirected graphs so that it runs in linear time, the data structure to be used is

- (A) Queue (B) Stack (C) Heap (D) B – Tree

32. Consider weighted complete graph  $G$  on the vertex set  $\{v_1, v_2, \dots, v_n\}$  such that the weight of the edge  $(v_i, v_j)$  is  $2|i - j|$ . The weight of a minimum spanning tree of  $G$  is

- (A)  $n - 1$  (B)  $2n - 2$  (C)  $\binom{n}{2}$  (D)  $n^2$

33. Let  $G$  be a directed graph whose vertex set is the set of number from 1 to 100. There is an edge from a vertex  $i$  to a vertex  $j$  iff either  $j = i + 1$  or  $j = 3i$ . the minimum number of edges in a path in  $G$  from vertex 1 to 100 is

- (A) 4 (B) 7 (C) 23 (D) 99

34. Let  $G$  be a weighted undirected graph and  $e$  be an edge with maximum weight in  $G$ . Suppose there is minimum weight spanning tree in  $G$  containing edge  $e$ . which of the following statements is always true?

- (A) There exists a cutset in  $G$  having all edges of maximum weight.  
(B) There exists a cycle in  $G$  having all edges of maximum weight.  
(C) Edge  $e$  can be contained in a cycle.  
(D) All edges in  $G$  have same weight.

35. In the following table, the left column contains the names of standard graph algorithms and the right column contains the time complexities of the algorithms. Match each algorithm with its time complexity.

1: Bellman-Ford algorithm	A: $O(m \log n)$
2: Kruskal's algorithm	B: $O(n^3)$
3: Floyd-Warshall algorithm	C: $O(nm)$
4: Topological sorting	D: $O(n + m)$

- (A)  $1 \rightarrow C, 2 \rightarrow A, 3 \rightarrow B, 4 \rightarrow D$   
(B)  $1 \rightarrow B, 2 \rightarrow D, 3 \rightarrow C, 4 \rightarrow A$   
(C)  $1 \rightarrow C, 2 \rightarrow D, 3 \rightarrow A, 4 \rightarrow B$   
(D)  $1 \rightarrow B, 2 \rightarrow A, 3 \rightarrow C, 4 \rightarrow D$

36. In depth first traversal of a graph  $G$  with  $n$  vertices,  $k$  edges are marked as tree edges. The number of connected components in  $G$  is

- (A)  $k$  (B)  $k + 1$  (C)  $n - k - 1$  (D)  $n - k$

37. **Statement for Linked Answer Questions 82a & 82b:**

Let  $s$  and  $t$  be two vertices in a undirected graph  $G = (V, E)$  having distinct positive edge weights. Let  $[X, Y]$  be partition of  $V$  such that  $s \in X$  and  $t \in Y$ . Consider the edge  $e$  having the minimum weight amongst all those edges that have one vertex in  $X$  and one vertex in  $Y$ .

- (a) The edge  $e$  must definitely belong to:
- (A) The minimum weighted spanning tree of  $G$   
(B) The weighted shortest path from  $s$  to  $t$   
(C) Each path from  $s$  to  $t$   
(D) The weighted longest path from  $s$  to  $t$

(b) Let the weight of an edge  $e$  denote the congestion on that edge. the congestion on a path is defined to be the maximum of the congestions on the edges of the path. We wish to find the path from  $s$  to  $t$  having minimum congestion. Which of the following paths is always such a path of minimum congestion?

- (A) A path from  $s$  to  $t$  in the minimum weighted spanning tree
- (B) A weighted shortest path from  $s$  to  $t$
- (C) An Euler walk from  $s$  to  $t$
- (D) A Hamiltonian path from  $s$  to  $t$

38. Postorder traversal of a given binary search tree,  $T$  produces the following sequence of keys

10, 9, 23, 22, 27, 25, 15, 50, 95, 60, 40, 29

Which one of the following sequences of keys can be the result of an inorder traversal of tree  $T$ ?

- (A) 9, 10, 15, 22, 23, 25, 27, 29, 40, 50, 60, 95
- (B) 9, 10, 15, 22, 40, 50, 60, 95, 23, 25, 27, 29
- (C) 29, 15, 9, 10, 25, 22, 23, 27, 40, 60, 50, 95
- (D) 95, 50, 60, 40, 27, 23, 22, 25, 10, 9, 15, 29

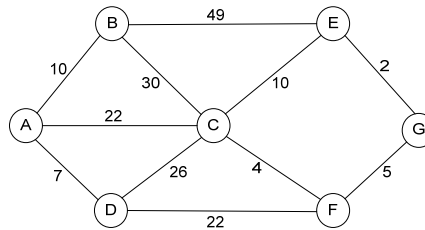
39. Let  $G$  be a simple connected graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is:

- (A) 6
- (B) 8
- (C) 9
- (D) 13

40. An undirected graph  $G$  has  $n$  nodes. its adjacency matrix is given by an  $n \times n$  square matrix whose (i) diagonal elements are 0's and (ii) non-diagonal elements are 1's. Which one of the following is TRUE?

- (A) Graph  $G$  has no minimum spanning tree (MST)
- (B) Graph  $G$  has a unique MST of cost  $n-1$
- (C) Graph  $G$  has multiple distinct MSTs, each of cost  $n-1$
- (D) Graph  $G$  has multiple minimum spanning trees of different costs.

41. Consider the undirected graph below.



Using Prim's algorithm to construct a minimum spanning tree starting with node  $A$ , which one of the following sequences of edges represents a possible order in which the edges would be added to construct the minimum spanning tree?

- (A) (E, G), (C, F), (F, G), (A, D), (A, B), (A, C)
- (B) (A, D), (A, B), (A, C), (C, F), (G, E), (F, G)
- (C) (A, B), (A, D), (D, F), (F, G), (G, E), (F, C)
- (D) (A, D), (A, B), (D, F), (F, C), (F, G), (G, E)

42. What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

- (A) 10
- (B) 11
- (C) 18
- (D) 19

43. What is the number of edges in an acyclic undirected graph with  $n$  vertices?

- (A)  $n - 1$

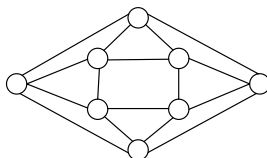
- (B)  $n$
- (C)  $n + 1$
- (D)  $2n - 2$

44. Let  $G_1 = (V, E_1)$  and  $G_2 = (V, E_2)$  be connected graphs on the same vertex set  $V$  with more than two vertices. If  $G_1 \cap G_2 = (V, E_1 \cap E_2)$  is not a connected graph, then the graph  $G_1 \cup G_2 = (V, E_1 \cup E_2)$
- (A) Cannot have a cut vertex
  - (B) Must have a cycle
  - (C) Must have a cut-edge (bridge)
  - (D) Has chromatic number strictly greater than those of  $G_1$  and  $G_2$

45. How many graphs on  $n$  labeled vertices exist which have at least  $(n^2 - 3n)/2$  edges?

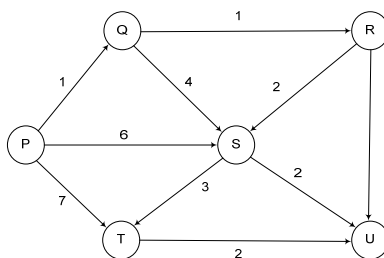
- (A)  $\binom{(n^2-3n)/2}{n} C_{(n^2-3n)/2}$
- (B)  $\sum_{k=0}^{(n^2-3n)/2} \binom{(n^2-n)}{k} C_k$
- (C)  $\binom{(n^2-n)/2}{n} C_n$
- (D)  $\sum_{k=0}^n \binom{(n^2-n)/2}{k} C_k$

46. The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same colour, is



- (A) 2
- (B) 3
- (C) 4
- (D) 5

47. Suppose we run Dijkstra's single source shortest path algorithm on the following edge-weighted directed graph with vertex P as the source.



In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized?

- (A) P, Q, R, S, T, U
- (B) P, Q, R, U, S, T
- (C) P, Q, R, U, T, S
- (D) P, Q, T, R, U, S

48. A binary tree with  $n > 1$  nodes has  $n_1$ ,  $n_2$  and  $n_3$  nodes of degree one, two and three respectively. The degree of a node is defined as the number of its neighbours.

$n_3$  can be expressed as:

- (A)  $n_1 + n_2 - 1$
- (B)  $n_1 - 2$
- (C)  $\left\lceil \frac{n_1 + n_2}{2} \right\rceil$
- (D)  $n_2 - 1$

Starting with the above tree, while there remains a node  $v$  of degree two in the tree, add an edge between the two neighbours of  $v$  and then



remove  $v$  from the tree. How many edges will remain at the end of the process?

- (A)  $2 * n_1 - 3$
- (B)  $n_2 + 2 * n_1 - 2$
- (C)  $n_3 - n_2$
- (D)  $n_2 + n_1 - 2$

49. Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequences list nodes in the order in which we could have encountered them in the search?

- (A) II and III only
- (B) I and III only
- (C) III and IV only
- (D) III only

Which of the following statements is TRUE?

- (A) I, II and IV are inorder sequences of three different BSTs
- (B) I is a preorder sequence of some BST with 439 as the root
- (C) II is an inorder sequence of some BST where 121 is the root and 52 is a leaf
- (D) IV is a postorder sequence of some BST with 149 as the root.

How many distinct BSTs can be constructed with 3 distinct keys?

- (A) 4
- (B) 5
- (C) 6
- (D) 9

50. In an unweighted, undirected connect graph, the shortest path from a node  $S$  to every node is computed most efficiently, in terms of *time complexity*, by

- (A) Dijkstra's algorithm starting from  $S$ .
- (B) Warshall's algorithm.
- (C) Performing a DFS starting from  $S$ .
- (D) Performing a BFS starting from  $S$ .

51. The following algorithm (written in pseudo-Pascal) works on a undirected graph  $G$

**Program Explore** ( $G$ )

**procedure** Visit ( $u$ )

**begin**

**if** Adj ( $u$ ) is not empty

{ comment : Adj ( $u$ ) is the list of edges incident to  $u$ }

**then**

**begin**

select an edge from Adj ( $u$ );

Let an edge  $e = (u, v)$ ;

remove  $e$  from Adj ( $u$ ) and Adj ( $v$ );

Visit ( $v$ );

**end**

**else**

mark  $u$  as a finished vertex and remove  $u$  from LIST;

{Comment : LIST is the set of vertices in the graph}

```

end;
begin
while LIST is not empty
do
begin
Let v ∈ LIST;
Visit (v);
end
end
end

```

NOTE: Initially Adj (u) is the list of all edges incident to u and LIST is the set of all vertices in the graph. They are globally accessible.

What kinds of sub graphs are obtained when this algorithm traverses the graphs G1 and G2 shown in Fig. 6 and Fig. 7, respectively?

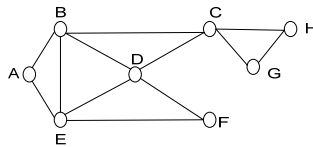


Fig. 6

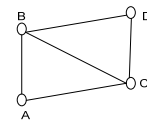


Fig. 7

- (a) What is the commonly known traversal of graphs that can be obtained from the sub graphs generated by *Program Explore*?
- (b) Show that the time complexity of the procedure is  $O(v + e)$  for a graph with  $v$  vertices and  $e$  edges, given that each vertex can be accessed and removed from LIST in const time. Also show that all edges of the graph are traversed.

52. The maximum number of possible edges in an undirected graph with  $n$  vertices and  $k$  components is \_\_\_\_\_.
53. Kruskal's algorithm for finding a minimum spanning tree of a weighted graph  $G$  with  $n$  vertices and  $m$  edges has the time-complexity of:
- $(n^2)$
  - $(m n)$
  - $(m + n)$
  - $(m \log n)$
  - $(m^2)$
54. Show that all vertices in an undirected finite graph cannot have distinct degrees, if the graph has at least two vertices
55. Complexity of Kruskal's algorithm for finding the minimum spanning tree of an undirected graph containing  $n$  vertices and  $m$  edges, if the edges are sorted is \_\_\_\_\_.
56. Maximum number of edges in a planar graph with  $n$  vertices is \_\_\_\_\_.
57. A non-planer graph with minimum number of vertices has
- 9 edges, 6 vertices
  - 6 edges, 4 vertices
  - 10 edges, 5 vertices
  - 9 edges, 5 vertices
58. How many edges are there in a forest with  $p$  components having  $n$  vertices in all?
59. An independent set in a graph is a subset of vertices such that no two vertices in the subset are connected by an edge. An incomplete scheme for greedy algorithm to find a maximum independent set in a tree is given below:
- V: = Set of all vertices in a tree;
- I: =  $\emptyset$  do

```

begin
    select a vertex  $u \in V$  such that
    _____;
     $V := V - \{u\}$ ;
    If  $u$  is such that
    _____ then  $I := I \cup \{u\}$ 
end;
Output (I);

```

Complete the algorithm by specifying the property of vertex  $u$  in each case. (4)

What is the time complexity of the algorithm?

60. Consider a simple connected graph  $G$  with  $n$  vertices and  $n$  edges ( $n > 2$ ). Then which of the following statements are true

- (f)  $G$  has no cycles
- (g) the graph obtained by removing any edges from  $G$  is not consider connected
- (h)  $G$  has at least one cycle
- (i) the graph obtained by removing any two edges from  $G$  is not consider connected
- (j) none of the above.

61. The number of distinct simple graphs with upto three nodes is

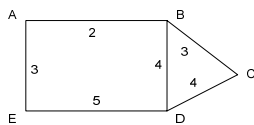
- (A) 15
- (B) 10
- (C) 7
- (D) 9

62. The number of edges in a regular graph of degree  $d$  and  $n$  vertices are .....

63. The minimum number of edges in a connected cyclic graph on  $n$  vertices is

- (A)  $n - 1$
- (B)  $n$
- (C)  $n + 1$
- (D) None of the above

64. How many minimum spanning trees does the following graph have? Draw them (weights are assigned to the edges).



65. Prove that in finite graph, the number of vertices of odd degree is always even.

66. A complete undirected, weighted graph  $G$  is given on the vertex set  $\{0, 1, \dots, n-1\}$  for any fixed ' $n$ '. Draw the minimum spanning tree of  $G$  if

- (a) The weight of the edge  $(u, v)$  is  $|u - v|$
- (b) The weight of the edge  $(u, v)$  is  $u + v$

67. Let  $G$  be the directed, weighted graph shown below in Fig. 17

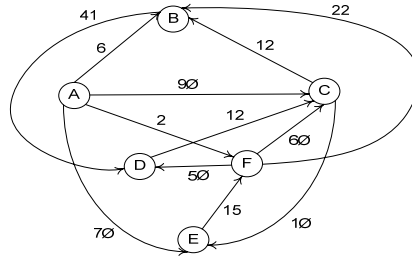


Fig. 17

We are interested in the shortest paths from A.

- (a) Output the sequence of vertices identified by the Dijkstra's algorithm for single source shortest path when the algorithm is started at node A. (2)
- (b) Write down the sequence of vertices in the shortest path from A to E. (2)
- (c) What is the cost of shortest path from A to E?

68. Which of the following algorithm design techniques is used in finding all pairs of shortest distances in a graph?

- (a) Dynamic programming
- (b) Backtracking
- (c) Greedy
- (d) Divide and Conquer

69. Consider a graph whose vertices are points in the plane with integer co-ordinates  $(x,y)$  such that  $1 \leq x \leq n$  and  $1 \leq y \leq n$ , where  $n \geq 2$  is an integer. Two vertices  $(x_1, y_1)$  and  $(x_2, y_2)$  are adjacent iff  $|x_1 - x_2| \leq 1$   $|y_1 - y_2| \leq 1$ . The weight of an edge  $\{(x_1, y_1), (x_2, y_2)\}$  is  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ .

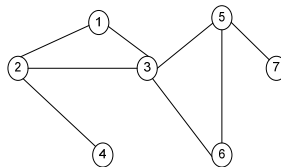
- (a) What is the weight of a minimum weight spanning tree in this graph? Write only the answer without any explanations. (2)

What is the weight of a maximum weight spanning tree in this graph? Write only the answer without any explanations.

70. Let G be a graph with 100 vertices numbered 1 to 100. Two vertices i and j are adjacent iff  $|i - j| = 8$  or  $|i - j| = 12$ . The number of connected components in G is

- (A) 8
- (B) 4
- (C) 12
- (D) 25

71. The number of articulation points of the following graph is:



- (A) 0
- (B) 1
- (C) 2
- (D) 3

72. Let G be an undirected graph with distinct edge weights. Let  $e_{max}$  be the edge with maximum weight and  $e_{min}$  the edge with minimum weight.

Which of the following is false?

- (A) Every minimum spanning tree of G must contain  $e_{min}$
- (B) If  $e_{max}$  is an minimum spanning tree, then its removal must disconnect G
- (C) No minimum spanning tree contains  $e_{max}$

(D) G has a unique minimum spanning tree

73. Let G be an undirected graph. Consider a depth first traversal G, and let T be the resulting depth first search tree. Let u be a vertex in G and v be the first new (unvisited) vertex visited after visiting u in the traversal. Which of the following is always true?

- (A) {u,v} must be an edge in G, and u is descendent of v in T
- (B) {u,v} must be an edge in G, and v is descendent of u in T
- (C) If {u,v} is not an edge in G then u is leaf in T
- (D) If {u,v} is not an edge in G then u and v must have the same parent in T

74. How many undirected graphs (not necessarily connected) can be constructed out of a given set  $And = \{v_1, v_2, \dots, v_n\}$  of n vertices?

- A.  $n(n-1)/2$
- B.  $2^n$
- C.  $n!$
- D.  $2^{n(n-1)/2}$

75. Consider a weighted undirected graph with vertex set  $And = \{n_1, n_2, n_3, n_4, n_5, n_6\}$  and edge set  $E = \{(n_1, n_2, 2), (n_1, n_3, 8), (n_1, n_6, 3), (n_2, n_4, 4), (n_2, n_5, 12), (n_3, n_4, 7), (n_4, n_5, 9), (n_4, n_6, 4)\}$  The third value in each tuple represents the weight of the edge specified in the tuple.

- (a) Lists the edges of a minimum spanning tree of the graph. (2)
- (b) How many distinct minimum spanning tree does this graph have?
- (c) Is the minimum among the edge weights of a minimum spanning tree unique over all possible minimum spanning trees of a graph? (1)
- (d) Is the maximum among the edge weights of a minimum spanning tree unique over all possible minimum spanning trees of a graph?

76. The minimum number of colors required to color the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same color is:

- A. 2
- B. 3
- C. 4
- D.  $n - 2 \lfloor n/2 \rfloor + 2$

77. The Maximum number of edges in a n -node undirected graph without self loops is

- A.  $n^2$
- B.  $n(n-1)/2$
- C.  $n-1$
- D.  $(n+1)(n)/2$

78. The number of distinct simple graphs with upto three nodes is

- (E) 15
- (F) 10
- (G) 7
- (H) 9

79. Fill in the blanks in the following template of an algorithm to compute all pairs shortest path lengths in a directed graph G with  $n \times n$  adjacency matrix A.  $A[i, j]$  equals 1 if there is an edge in G from i to j, and 0 otherwise. Your aim in filling in the blanks

INITIALIZATION: For  $i = 1 \dots n$

{ For  $j = 1 \dots n$

{ If  $A[i, j] = 0$  then  $P[i, j] = \underline{\hspace{2cm}}$  else  $P[i, j] = \underline{\hspace{2cm}};$

}

ALGORITHM: For  $i = 1 \dots n$

{ For  $j = 1 \dots n$

```

    { For k = 1 ... n
      {P[_,_] = min{ _____ };}
    }
  }

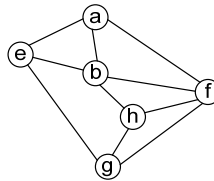
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- (a) Copy the complete line containing the blanks in the Initialization step and fill in the blanks. (1)  
 (b) Copy the complete line containing the blanks in the Initialization step and fill in the blanks. (3)  
 (c) Fill in the blank: The running time of the algorithm is  $O(\text{_____})$

80. Let  $G$  be an arbitrary graph with  $n$  nodes and  $k$  components. If a vertex is removed from  $G$ , the number of components in the resultant graph must necessarily lie between

- (A)  $k$  and  $n$   
 (B)  $k - 1$  and  $k + 1$   
 (C)  $k - 1$  and  $n - 1$   
 (D)  $k + 1$  and  $n - k$

81. Consider the following graph



Among the following sequences

- I a b e g h f  
 II a b f e h g  
 III a b f h g e  
 IV a f g h b e

Which are depth first traversals of the above graph?

- (A) I, II and IV only  
 (B) I and IV only  
 (C) II, III and IV only  
 (D) I, III and IV only

82. How many perfect matching are there in a complete graph of 6 vertices?

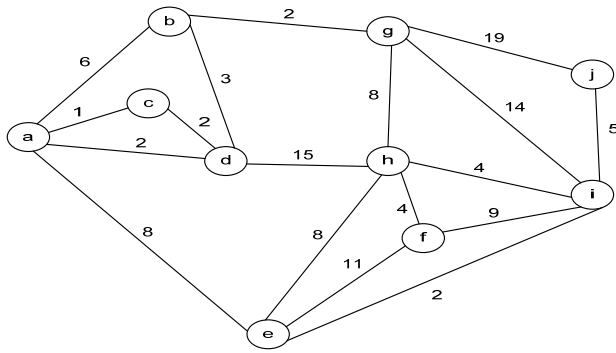
- (A) 15  
 (B) 24  
 (C) 24  
 (D) 60

83. Let  $G = (V, E)$  be an undirected graph with a subgraph  $G_1 = (V_1, E_1)$ . Weights are assigned to edges of  $G$  as follows.

A single-source shortest path algorithm is executed on the weighted graph  $(V, E, \text{weight})$  with an arbitrary vertex  $v_1$  of  $V_1$  as the source. Which of the following can always be inferred from the path costs computed?

- (A) The number of edges in the shortest paths from  $v_1$  to all vertices of  $G$   
 (B)  $G_1$  is connected.  
 (C)  $V_1$  forms a clique in  $G$   
 (D)  $G_1$  is a tree

84. What is the weight of a minimum spanning tree of the following graph?



- (A) 29
- (B) 31
- (C) 38
- (D) 41