1 Common part

1. Let X be a set with n elements. How many subsets of X have odd cardinality?

(a) n

(b) 2^n

(c) $2^{n/2}$

(d) 2^{n-1} ✓

(e) Cannot be determined without knowing whether n is odd or even

2. How many proper divisors (that is, divisors other than 1 or 7200) does 7200 have?

(a) 18

(b) 20

(c) 52 ✓

(d) 54

(e) 60

3. A is an $n \times n$ square matrix for which the entries in every row sum to 1. Consider the following statements:

(i) The column vector $[1, 1, \ldots, 1]^T$ is an eigenvector of A.

(ii) $\det(A - I) = 0$.

(iii) $\det(A) = 0$.

Which of the above statements must be TRUE?

(a) Only (i)

(b) Only (ii)

(c) Only (i) and (ii) ✓

(d) Only (i) and (iii)

(e) (i), (ii) and (iii)

4. What is the probability that a point $P = (\alpha, \beta)$ picked uniformly at random from the disk $x^2 + y^2 \leq 1$ satisfies $\alpha + \beta \leq 1$?

(a) $\frac{1}{\pi}$

(b) $\frac{3}{4} + \frac{1}{4} \cdot \frac{1}{\pi}$

(c) $\frac{3}{4} + \frac{1}{4} \cdot \frac{2}{\pi}$ ✓

(d) 1

(e) $\frac{2}{\pi}$

5. Asha and Lata play a game in which Lata first thinks of a natural number between 1 and 1000. Asha must find out that number by asking Lata questions, but Lata can only reply by saying “yes” or “no”. Assume that Lata always tells the truth. What is the least number of questions that Asha needs to ask within which she can always find out the number Lata has thought of?

(a) 10 ✓

(b) 32

(c) 100

(d) 999

(e) None of the above

6. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be *convex* if for all $x, y \in \mathbb{R}$ and λ such that $0 \leq \lambda \leq 1$,

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y).$$

Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function, and define the following functions:

$$p(x) = f(-x), \quad q(x) = -f(-x), \text{ and } r(x) = f(1 - x).$$

Which of the functions p, q and r must be convex?

(a) Only p

(b) Only q

(c) Only r

(d) Only p and r ✓

(e) Only q and r

7. What are the last two digits of $1! + 2! + \cdots + 100!$?

(a) 00

(b) 13 ✓

(c) 30

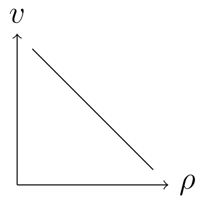
(d) 33

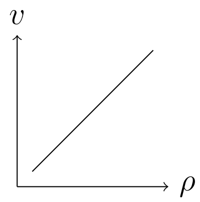
(e) 73

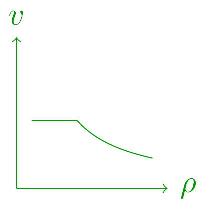
8. Consider the following toy model of traffic on a straight, single lane, highway. We think of cars as points, which move at the maximum speed v that satisfies the following constraints:

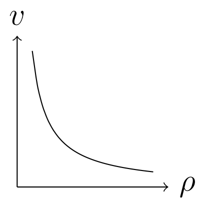
1. The speed is no more than the speed limit $v_{\max}$ mandated for the highway.
2. The speed is such that when travelling at this speed, it takes at least time t_0 (where t_0 is a fixed time representing the reaction time of drivers) to reach the car ahead, in case the car ahead stops suddenly.

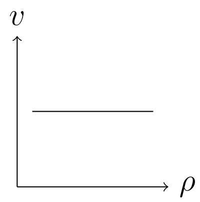
Let as assume that in the steady state, all cars on the highway move at the same speed v satisfying both the above constraints, and the distance between any two successive cars is the same. Let ρ denote the "density", that is, the number of cars per unit length of the highway. Which of the following graphs most accurately captures the relationship between the speed v and density ρ in this model?

(a) 

(b) 

(c)  ✓

(d) 

(e) 

9. Let A and B be two containers. Container A contains 50 litres of liquid X and container B contains 100 litres of liquid Y. Liquids X and Y are soluble in each other.

We now take 30ml of liquid X from container A and put it into container B. The mixture in container B is then thoroughly mixed and 20ml of the resulting mixture is put back into container A. At the end of this process let V_{AY} be the volume of liquid Y in container A and V_{BX} be the volume of liquid X in container B. Which of the following must be TRUE?

(a) $V_{AY} < V_{BX}$ ✓

(b) $V_{AY} > V_{BX}$

(c) $V_{AY} = V_{BX}$

(d) $V_{AY} + V_{BX} = 30$

(e) $V_{AY} + V_{BX} = 20$

10. Avni and Badal alternately choose numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ without replacement (starting with Avni). The first person to choose numbers of which any 3 sum to 15 wins the game (for example, Avni wins if she chooses the numbers 8, 3, 5, 2, since $8 + 5 + 2 = 15$). A player is said to have a winning strategy if the player can always win the game, no matter what the other player does. Which of the following statements is TRUE?

As a hint, there are exactly 8 ways in which 3 numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ can sum up to 15, shown as the three rows, the three columns, and the two diagonals in the following square:

$$\begin{array}{ccc} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{array}$$

(a) Avni has a winning strategy

(b) Badal has a winning strategy

(c) Both of them have a winning strategy

(d) Neither of them has a winning strategy ✓

(e) The player that picks 9 has a winning strategy

11. Suppose there are n guests at a party (and no hosts). As the night progresses, the guests meet each other and shake hands. The same pair of guests might shake hands multiple times, for some parties stretch late into the night, and it is hard to keep track. Still, they don't shake hands with themselves. Let Odd be the set of guests who have shaken an odd number of hands, and let Even be the set of guests who have shaken an even number of hands. Which of the following stays invariant throughout the night?

(a) $|\mathsf{Odd}| \bmod 2$ ✓

(b) $|\mathsf{Even}|$

(c) $|\mathsf{Even}| \cdot |\mathsf{Odd}|$

(d) $2|\mathsf{Even}| - |\mathsf{Odd}|$

(e) $2|\mathsf{Odd}| - |\mathsf{Even}|$

12. Let f be a function with both input and output in the set $\{0, 1, 2, \ldots, 9\}$, and let the function g be defined as $g(x) = f(9 - x)$. The function f is nondecreasing, so that $f(x) \geq f(y)$ for $x \geq y$. Consider the following statements:

(i) There exists $x \in \{0, \ldots, 9\}$ so that $x = f(x)$.

(ii) There exists $x \in \{0, \ldots, 9\}$ so that $x = g(x)$.

(iii) There exists $x \in \{0, \ldots, 9\}$ so that $x = (f(x) + g(x)) \bmod 10$.

Which of the above statements must be TRUE for ALL such functions f and g?

(a) Only (i)

(b) Only (i) and (ii) ✓

(c) Only (iii)

(d) None of them

(e) All of them

13. Consider the integral

$$\int_0^1 \frac{x^{300}}{1+x^2+x^3}dx.$$

What is the value of this integral correct up to two decimal places?

(a) 0.00 ✓

(b) 0.02

(c) 0.10

(d) 0.33

(e) 1.00

14. A drawer contains 9 pens, of which 3 are red, 3 are blue, and 3 are green. The nine pens are drawn from the drawer one at a time (without replacement) such that each pen is drawn with equal probability from the remaining pens in the drawer. What is the probability that two red pens are drawn in succession?

(a) 7/12 ✓

(b) 1/6

(c) 1/12

(d) 1/81

(e) None of the above

15. Consider the matrix

$$A = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 3/4 & 1/4 \\ 0 & 1/4 & 3/4 \end{bmatrix}.$$

What is $\lim_{n\to\infty} A^n$?

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1/4 & 1/2 & 1/2 \\ 1/4 & 1/2 & 1/2 \\ 1/4 & 1/2 & 1/2 \end{bmatrix}$

(c) $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ ✓

(e) The limit exists, but it is none of the above

2 Computer Science

1. Which of the following decimal numbers can be exactly represented in binary notation with a finite number of bits?

(a) 0.1

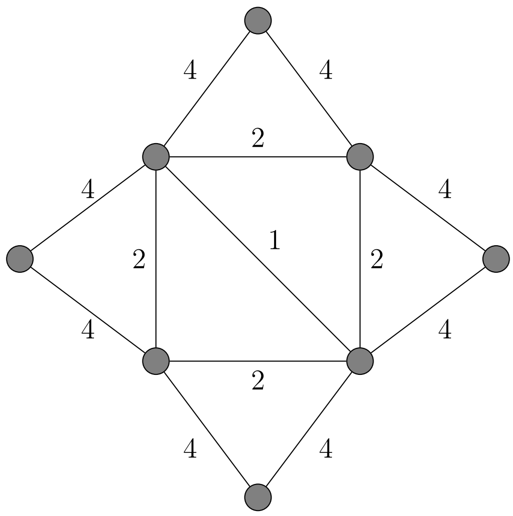
(b) 0.2

(c) 0.4

(d) 0.5 ✓

(e) All the above

2. How many distinct minimum weight spanning trees does the following undirected, weighted graph have?



(a) 8

(b) 16

(c) 32

(d) 64 ✓

(e) None of the above

3. A graph is d-regular if every vertex has degree d. For a d-regular graph on n vertices, which of the following must be TRUE?

(a) d divides n

(b) Both d and n are even

(c) Both d and n are odd

(d) At least one of d and n is odd

(e) At least one of d and n is even ✓

4. Let φ be a propositional formula on a set of variables A and ψ be a propositional formula on a set of variables B, such that $\varphi \Rightarrow \psi$. A *Craig interpolant* of φ and ψ is a propositional formula μ on variables $A \cap B$ such that $\varphi \Rightarrow \mu$ and $\mu \Rightarrow \psi$.

Given propositional formula $\varphi = q \vee (r \wedge s)$ on the set of variables $A = \{q, r, s\}$ and propositional formula $\psi = \neg q \rightarrow (s \vee t)$ on the set of variables $B = \{q, s, t\}$, which of the following is a Craig interpolant for φ and ψ?

(a) q

(b) φ itself

(c) $q \vee s$ ✓

(d) $q \vee r$

(e) $\neg q \wedge s$

5. Stirling's approximation for $n!$ states that for some constants c_1, c_2,

$$c_1\, n^{n+\frac{1}{2}}\, e^{-n} \;\leq\; n! \;\leq\; c_2\, n^{n+\frac{1}{2}}\, e^{-n}\,.$$

What are the tightest asymptotic bounds that can be placed on $n!$?

(a) $n! = \Omega\left(n^n\right)$ and $n! = O\left(n^{n+\frac{1}{2}}\right)$

(b) $n! = \Theta\left(n^{n+\frac{1}{2}}\right)$

(c) $n! = \Theta\left(\left(\frac{n}{e}\right)^n\right)$

(d) $n! = \Theta\left(\left(\frac{n}{e}\right)^{n+\frac{1}{2}}\right)$ ✓

(e) $n! = \Theta\left(n^{n+\frac{1}{2}} 2^{-n}\right)$

6. Given the following pseudocode for function `printx()` below, how many times is `x` printed if we execute `printx(5)`?

```
void printx(int n) {
    if (n==0) {
        printf("x");
    }
    for (int i=0;i<=n-1;++i){
        printx(n-1);
    }
}
```

(a) 625

(b) 256

(c) 120 ✓

(d) 24

(e) 5

7. A formula is said to be a 3-CF-formula if it is a conjunction (i.e., an AND) of clauses, and each clause has at most 3 literals. Analogously, a formula is said to be a 3-DF-formula if it is a disjunction (i.e., an OR) of clauses of at most 3 literals each.

Define the languages 3-CF-SAT and 3-DF-SAT as follows:

$$\text{3-CF-SAT} = \{\Phi | \Phi \text{ is a } \textit{satisfiable} \text{ 3-CF formula}\}$$
$$\text{3-DF-SAT} = \{\Phi | \Phi \text{ is a } \textit{satisfiable} \text{ 3-DF formula}\}$$

Which of the following best represents our current knowledge of these languages?

(a) Both 3-CF-SAT and 3-DF-SAT are in NP but only 3-CF-SAT is NP-complete ✓

(b) Both 3-CF-SAT and 3-DF-SAT are NP-complete

(c) Both 3-CF-SAT and 3-DF-SAT are in P

(d) Both 3-CF-SAT and 3-DF-SAT are in NP but only 3-DF-SAT is NP-complete

(e) Neither 3-CF-SAT nor 3-DF-SAT are in P

8. Consider the following program fragment:

```
var a, b : integer ;
procedure G (c, d : integer) ;
begin
    c := c - d ;
    d := c + d ;
    c := d - c
end ;
a := 2 ;
b := 3 ;
G (a, b);
```

If both parameters to `G` are passed by reference, what are the values of a and b at the end of the above program fragment?

(a) $a = 0$ and $b = 2$

(b) $a = 3$ and $b = 2$ ✓

(c) $a = 2$ and $b = 3$

(d) $a = 1$ and $b = 5$

(e) None of the above

9. Consider the following program fragment:

```
var x, y: integer ;
x := 1; y := 0 ;
while y < x do
```

```
begin
    x := 2 * x ;
    y := y + 1
end ;
```

For the above fragment, which of the following is a loop invariant?

(a) $x = y + 1$

(b) $x = (y + 1)^2$

(c) $x = (y + 1)2^y$

(d) $x = 2^y$ ✓

(e) None of the above, since the loop does not terminate

10. Let the language D be defined on the binary alphabet $\{0, 1\}$ as follows:

$$D := \{w \in \{0,1\}^* | \text{ substrings 01 and 10 occur an equal number of times in } w\}.$$

For example, $101 \in D$ while $1010 \notin D$. Which of the following must be TRUE of the language D?

(a) D is regular ✓

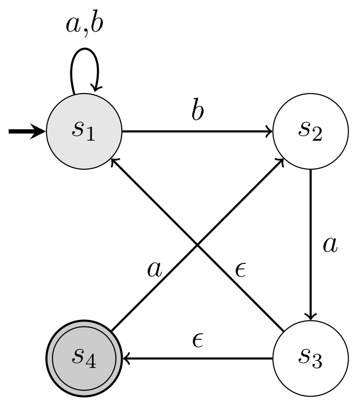
(b) D is context-free but not regular

(c) D is decidable but not context-free

(d) D is decidable but not in NP

(e) D is undecidable

11. Consider the following non-deterministic automaton, where s_1 is the start state and s_4 is the final (accepting) state. The alphabet is $\{a, b\}$. A transition with label ϵ can be taken without consuming any symbol from the input.



Which of the following regular expressions corresponds to the language accepted by this automaton?

(a) $(a + b)^*aba$

(b) $(a + b)^*ba^*$

(c) $(a+b)^*ba(aa)^*$ ✓

(d) $(a+b)^*$

(e) $(a+b)^*baa^*$

12. Let $G = (V, E)$ be a directed graph with n (≥ 2) vertices, including a special vertex r. Each edge $e \in E$ has a strictly positive edge weight $w(e)$. An arborescence in G rooted at r is a subgraph H of G in which every vertex $u \in V \setminus \{r\}$ has a directed path to the special vertex r. The weight of an arborescence H is the sum of the weights of the edges in H.

Let H^* be a minimum weight arborescence rooted at r, and w^* the weight of H^*. Which of the following is NOT always true?

(a)

$$w^* \geq \sum_{u \in V \setminus \{r\}} \min_{(u,v) \in E} w((u,v))$$

(b)

$$w^* \geq \sum_{u \in V \setminus \{r\}} \min_{(v,u) \in E} w((v,u))$$

✓

(c) H^* has exactly $n-1$ edges

(d) H^* is acyclic

(e) w^* is less than the weight of the minimum weight directed Hamiltonian cycle in G, whenever G has a directed Hamiltonian cycle

13. A row of 10 houses has to be painted using the colours red, blue, and green so that each house is a single colour, and any house that is immediately to the right of a red or a blue house must be green. How many ways are there to paint the houses?

(a) 199

(b) 683

(c) 1365 ✓

(d) $3^{10} - 2^{10}$

(e) 3^{10}

14. Let m and n be two positive integers. Which of the following is NOT always true?

(a) If m and n are co-prime, there exist integers a and b such that $am + bn = 1$

(b) $m^{n-1} \equiv 1 \pmod{n}$ ✓

(c) The rational number $\frac{n}{m} \cdot \frac{n-1}{m-1} \cdot \frac{n-2}{m-2} \cdots \frac{n-(m-2)}{m-(m-2)} \cdot \frac{n-(m-1)}{m-(m-1)}$ is an integer

(d) $m+1$ is a factor of $m^{n(n+1)} - 1$

(e) If $2^n - 1$ is prime, then n is prime

15. Consider directed graphs on n labelled vertices $\{1, 2, \ldots, n\}$, where each vertex has exactly one edge coming in and exactly one edge going out. We allow self-loops. How many such graphs have exactly two cycles?

(a) $\displaystyle\sum_{k=1}^{n-1} k!\,(n-k)!$

(b) $\displaystyle\frac{n!}{2}\left[\sum_{k=1}^{n-1} \frac{1}{k(n-k)}\right]$ ✓

(c) $\displaystyle n!\left[\sum_{k=1}^{n-1} \frac{1}{k}\right]$

(d) $\displaystyle\frac{n!\,(n-1)}{2}$

(e) None of the above

3 System Science

1. Consider a discrete-time system which in response to input sequence $x[n]$ (n integer) outputs the sequence $y[n]$ such that

$$y[n] = \begin{cases} 0, & n = -1, -2, -3, \dots, \\ \alpha y[2n-1] + \beta y[n-1] + \gamma x[n-1] + x[n] + 1, & n = 0, 1, 2, \dots \end{cases}$$

Which of the below makes the system linear and time-invariant?

(a) Only $\alpha = \beta = \gamma = 0$

(b) Only $\alpha = \beta = 0$ (parameter γ can take any value)

(c) Only $\alpha = 0$ (parameters β and γ can take arbitrary values)

(d) Always non-linear, but time-invariant only if $\alpha = 0$ (parameters β and γ can take arbitrary values) ✓

(e) Cannot be determined from the information given

2. Let A and B be two square matrices that have full rank. Let λ_A be an eignevalue of A and λ_B an eigenvalue of B. Which of the following is always TRUE?

(a) AB has full rank ✓

(b) $A - B$ has full rank

(c) $\lambda_A \lambda_B$ is an eigenvalue of AB

(d) $A + B$ has full rank

(e) At least one of λ_A or λ_B is an eigenvalue of AB

3. Consider a function $f : \mathbf{R} \to \mathbf{R}$ such that $f(x) = 1$ if x is rational, and $f(x) = 1 - \epsilon$, where $0 < \epsilon < 1$, if x is irrational. Which of the following is TRUE?

(a) $\lim_{x\to\infty} f(x) = 1$

(b) $\lim_{x\to\infty} f(x) = 1 - \epsilon$

(c) $\lim_{x\to\infty} f(x)$ exists, but is neither 1 nor $1 - \epsilon$

(d) $\max_{x \geq 1} f(x) = 1$ ✓

(e) None of the above

4. Let $f(x) = \sqrt{x^2 - 4x + 4}$, for $x \in (-\infty, \infty)$. Here, $\sqrt{y}$ denotes the non-negative square root of y when y is non-negative. Then, which of the following is TRUE?

(a) $f(x)$ is not continuous but differentiable

(b) $f(x)$ is continuous and differentiable

(c) $f(x)$ is continuous but not differentiable ✓

(d) $f(x)$ is neither continuous nor differentiable

(e) None of the above

5. Consider the function $f(x) = e^{x^2} - 8x^2$ for all x on the real line. For how many distinct values of x do we have $f(x) = 0$?

(a) 1

(b) 4 ✓

(c) 2

(d) 3

(e) 5

6. Suppose that X_1 and X_2 denote the random outcomes of independent rolls of two dice. Each of the dice takes each of the six values $1, 2, 3, 4, 5,$ and 6 with equal probability. What is the value of the conditional expectation

$$\mathbf{E}\left[\max(X_1, X_2) | \min(X_1, X_2) = 3\right]?$$

(a) 33/7 ✓

(b) 4

(c) 5

(d) 9/2

(e) 19/4

7. Consider two random variables X and Y which take values in a finite set S. Let $p_{X,Y}$ represent their joint probability mass function (p.m.f.) and let p_X and p_Y, respectively, be the marginal p.m.f.'s of X and Y, respectively. Which of the choices below is always equal to

$$\max_{A \subseteq S} |\Pr(X \in A) - \Pr(Y \in A)|?$$

(a) $\frac{1}{2} \sum_{n \in S} |p_X(n) - p_Y(n)|$ ✓

(b) $\frac{1}{2} \sum_{n_1, n_2 \in S : n_1 \neq n_2} p_{X,Y}(n_1, n_2)$

(c) $\frac{1}{2} \sum_{n \in S} p_{X,Y}(n, n)$

(d) $\frac{1}{2} \sum_{n_1, n_2 \in S : n_1 \neq n_2} |p_X(n_1) - p_Y(n_2)|$

(e) None of the above

8. Let K be a cube of side 1 in $\mathbb{R}^3$, with its centre at the origin, and its sides parallel to the co-ordinate axes. For $t \geq 0$, let K_t be the set of all points in $\mathbb{R}^3$ whose Euclidean distance to K is less than or equal to t. Let V_t be the volume of K_t. Then, which of the following is TRUE for all $t \geq 0$?

Note: If $d(x, y)$ denotes the Euclidean distance between two points x and y in $\mathbb{R}^3$, then the distance of a point $p \in \mathbb{R}^3$ to K is defined as the minimum of the quantities $d(p, y)$ when y ranges over K.

(a) $V_t \leq 1$

(b) $V \leq 1 + \frac{4}{3}\pi t^3$

(c) $V \leq 1 + \frac{4}{3}\pi t$

(d) $V \leq \frac{4}{3}\pi \left(\frac{\sqrt{3}}{2} + t\right)^3$ ✓

(e) $V \geq (1 + 2t)^3$

9. Consider a coin which comes up heads with probability p and tails with probability $1 - p$, where $0 < p < 1$. Suppose we keep tossing the coin until we have seen both sides of the coin. What is the expected number of times we would have seen tails? (Hint: the expected number of tosses required to see heads for the first time is 1/p.)

(a) $\frac{1}{p}$

(b) $1 + \frac{1}{1-p}$

(c) $p + \frac{1}{p} - 1$ ✓

(d) 2

(e) None of the above

10. Let X, Z_1, and Z_2 be independent random variables taking values in the set $\{0, 1\}$. X is uniformly distributed in $\{0, 1\}$, while the distributions of Z_1 and Z_2 are such that if we define $Y_1 = X + Z_1$ and $Y_2 = X + Z_2$, where addition is modulo 2, then

$$\Pr(Y_1 = 1|X = 0) = \Pr(Y_1 = 0|X = 1) = p_1, \text{ and}$$
$$\Pr(Y_2 = 1|X = 0) = \Pr(Y_2 = 0|X = 1) = p_2.$$

Consider the optimal estimator of X from observations of Y_1 and Y_2, defined by the following optimization problem:

$$\min_{f(.,.)} \Pr(X \neq f(Y_1, Y_2)),$$

where the minimization is over all functions f which map a pair of observation bits to an estimate bit. What is the value of the above minimum? You can assume that $p_1, p_2 \leq 1/2$.

(a) $\max(p_1, p_2)$

(b) $\min(p_1, p_2)$ ✓

(c) $(1/p_1 + 1/p_2)^{-1}$

(d) $(1 + 1/p_1 + 1/p_2)^{-1}$

(e) None of the above

11. Let X and Y be independent Gaussian random variables with means 1 and 2 and variances 3 and 4 respectively. What is the minimum possible value of $\mathbf{E}\left[(X + Y - t)^2\right]$, when t varies over all real numbers?

(a) 7 ✓

(b) 5

(c) 1.5

(d) 3.5

(e) 2.5

12. Consider an urn with a red and b blue balls. Balls are drawn out one-by-one, without replacement and uniformly at random, until the first red ball is drawn. What is the expected total number of balls drawn by this process? (Hint: Consider deriving an appropriate recurrence.)

(a) $\frac{a+b}{a+1}$

(b) $\frac{a+b+1}{a}$

(c) $\frac{a+b}{a}$

(d) $\frac{a+b+1}{a+1}$ ✓

(e) a

13. For $t > 0$, let S_t denote the ball of radius t centered at the origin in $\mathbb{R}^n$. That is,

$$S_t = \left\{ \mathbf{x} \in \mathbb{R}^n \;\middle|\; \sum_{i=1}^{n} x_i^2 \leq t^2 \right\}.$$

Let N_t be the number of points in S_t that have integer coordinates, and let V_t be the volume of S_t. Which of the following is TRUE?

(a) For any $t > 0$, N_t is less than the volume of S_t

(b) $\lim_{t\to\infty} \frac{N_t}{V_t} = \frac{1}{2}$

(c) $\lim_{t\to\infty} \frac{N_t}{V_t} = 2$

(d) $\lim_{t\to\infty} \frac{N_t}{V_t} = 1$ ✓

(e) N_t is a monotonically decreasing function of t

14. Consider the circle of radius 1 centred at the origin in two dimensions. Choose two points x and y independently at random so that both are uniformly distributed on the circle. Let the vectors joining the origin to x and y be X and Y, respectively. Let θ be the angle between X and Y, measured in an anti-clockwise direction while moving along the circle from x towards y. Which of the following is TRUE?

(a) $\mathbf{E}[\theta] = \pi$ ✓

(b) $\mathbf{E}[|x-y|^2] = \sqrt{2}$

(c) $\mathbf{E}[|x-y|^2] = 1 + \sqrt{2}$

(d) $\mathbf{E}[|x-y|^2] = \sqrt{3}$

(e) $\mathbf{E}[|x-y|^2] = 1$

15. Anu reached a bus stop at 9:00 AM. She knows that the number of minutes after 9:00 AM when the bus will arrive is distributed with probability density function (p.d.f.) f where

$$f(x) = \frac{1}{10}\exp(-x/10)$$

for $x \geq 0$, and $f(x) = 0$ for $x < 0$.

At 9:05 AM, the bus had still not arrived. Given her knowledge of this fact and of the p.d.f. f of the arrival time of the bus, at what time, measured in minutes after 9:00 AM, would Anu expect the bus to arrive?

(a) 12.5

(b) 15 ✓

(c) 7.5

(d) 10

(e) 12.5